Dual-Priced Parametric Modal Transition Systems

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Copenhagen Meeting on Variability Analysis, ITU, 18.11.2013

Joint work with Nikola Beneš, Jan Křetínský, Kim G. Larsen and Mikael H. Møller.

Motivation

Overall Aim

Need for a sound theory supporting a step-wise, component-based design of a software system.

- Components are specified in a formal way at a certain abstraction level.
- Specifications are gradually refined until a concrete system is produced.
- If the refinement steps preserve certain properties, the final system will as well.

Motivation

We focus on Modal Transition Systems (MTS), a specification formalism introduced by K.G. Larsen and B. Thomsen.

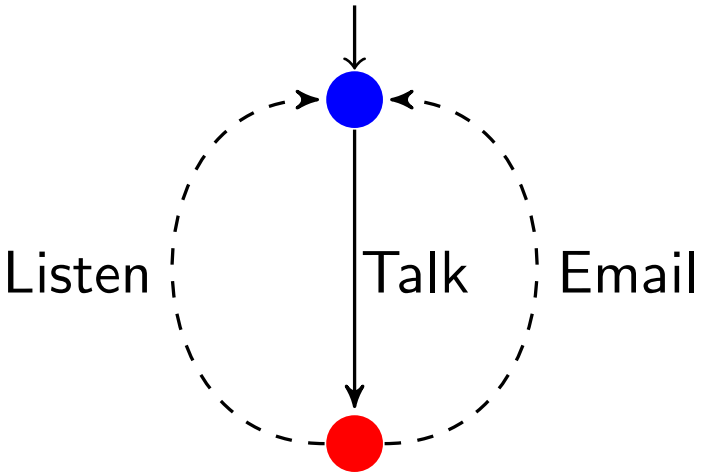
- MTS are automata-based specification formalism.
- MTS allow to express that certain actions must or may happen in their implementations.
- Recently many applications in component-based software development, interface theories, modal abstractions and program analysis.

Outline of the Talk

1. MTS definition, refinement relations, basic results.
2. Parametric MTS.
3. Dual-Priced MTS.

Example: ITU Meeting

Specification:



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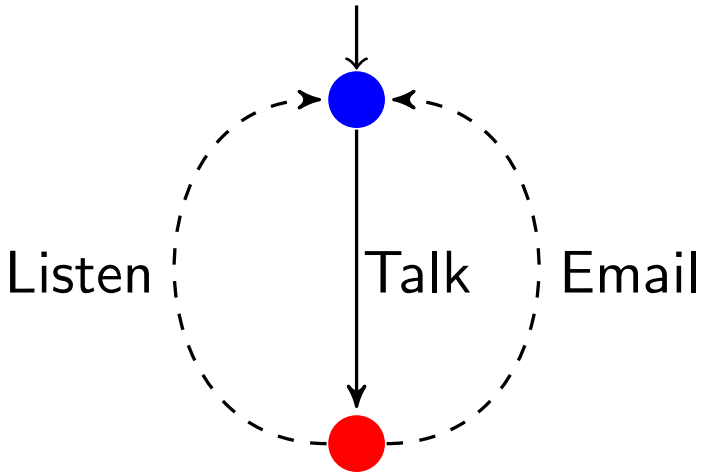
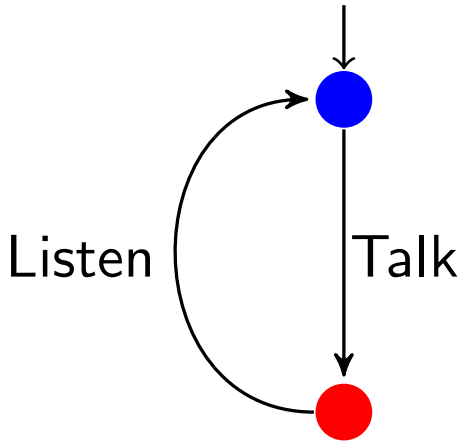


Implementations:



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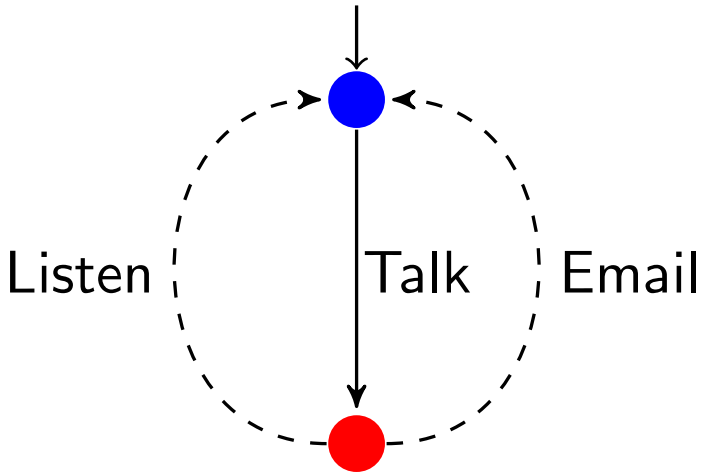

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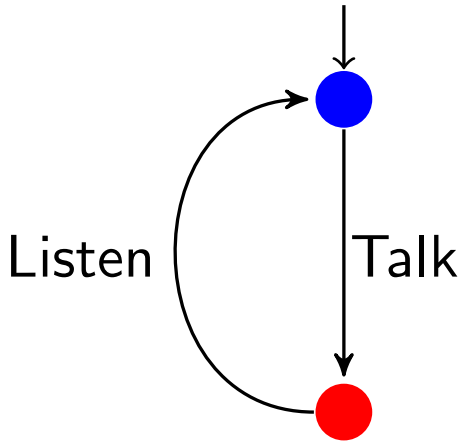

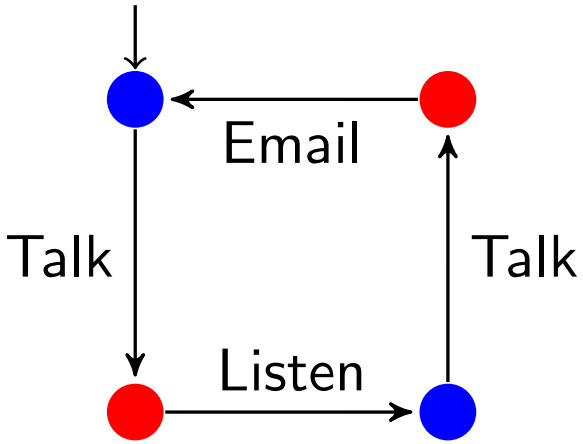

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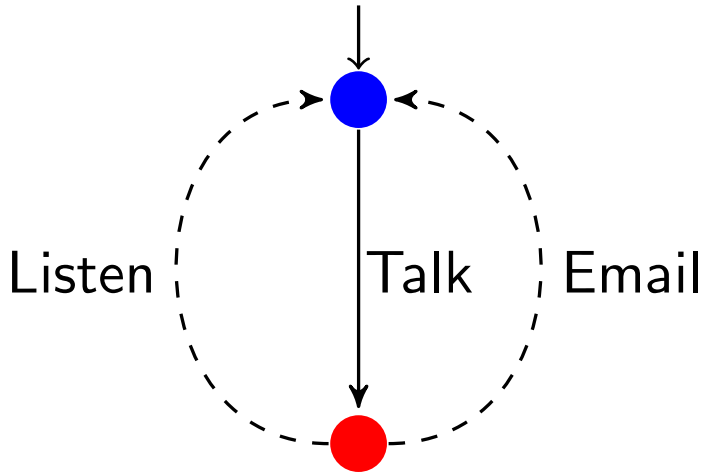

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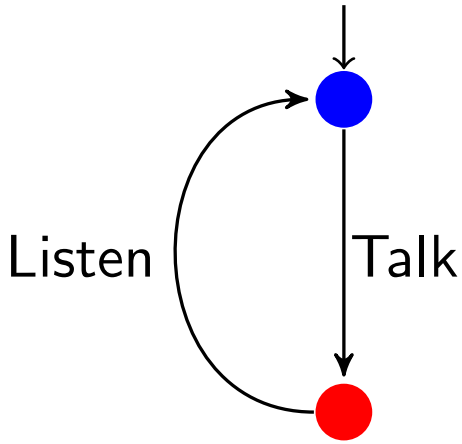
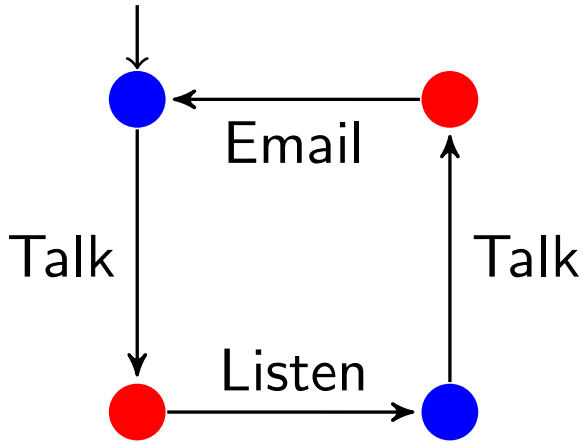
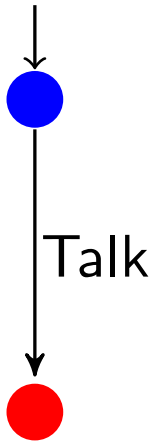
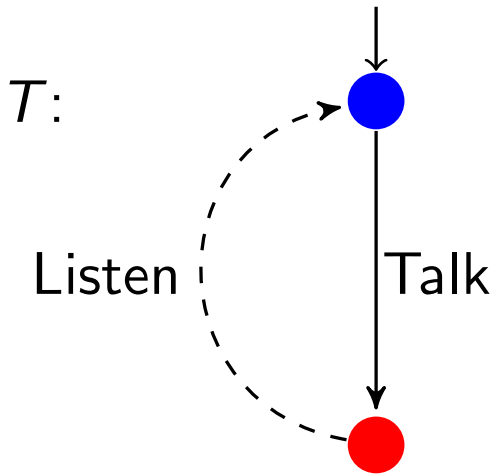
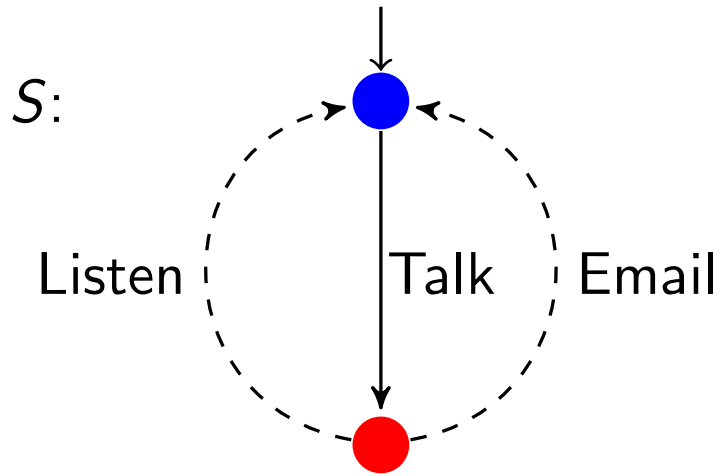






Modal Refinement





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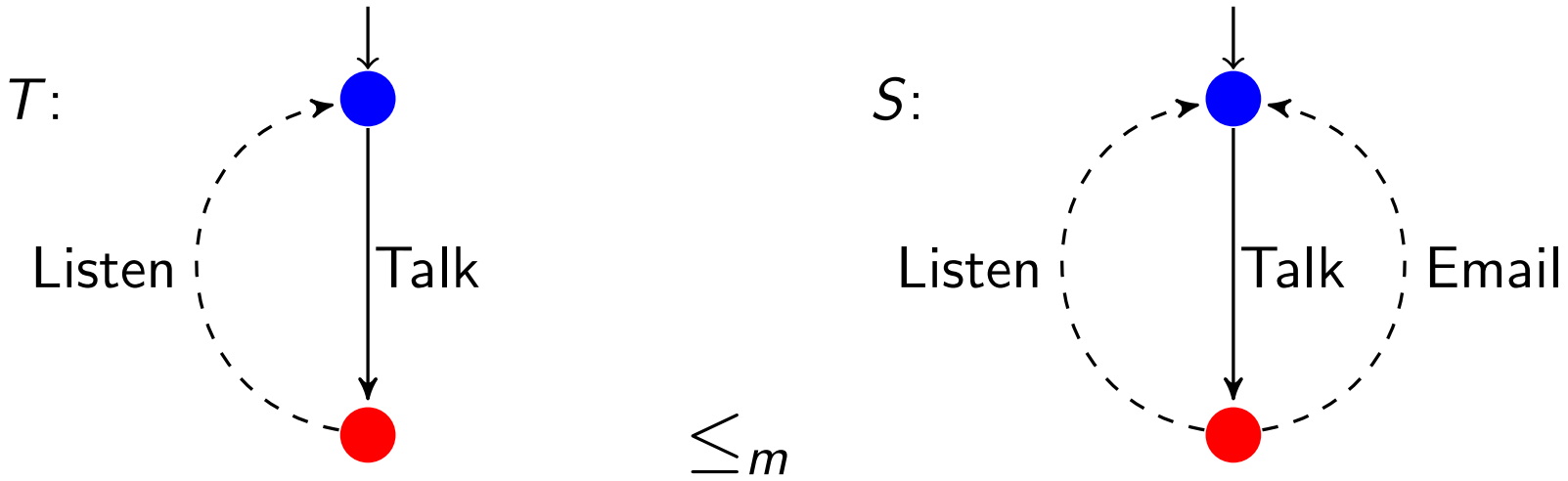

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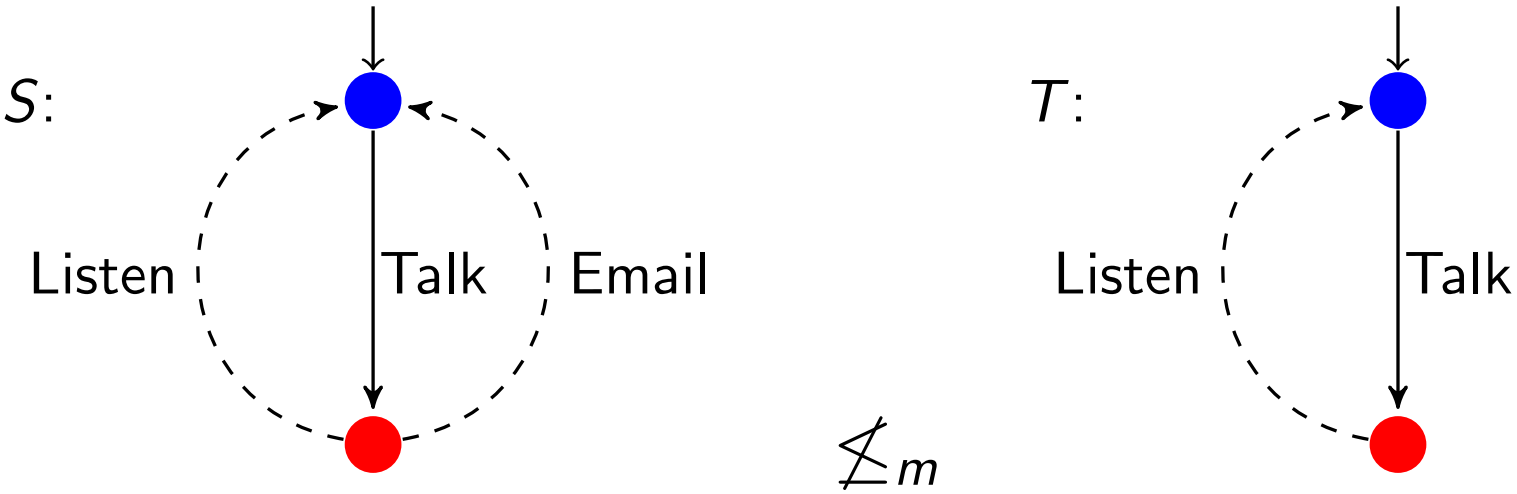

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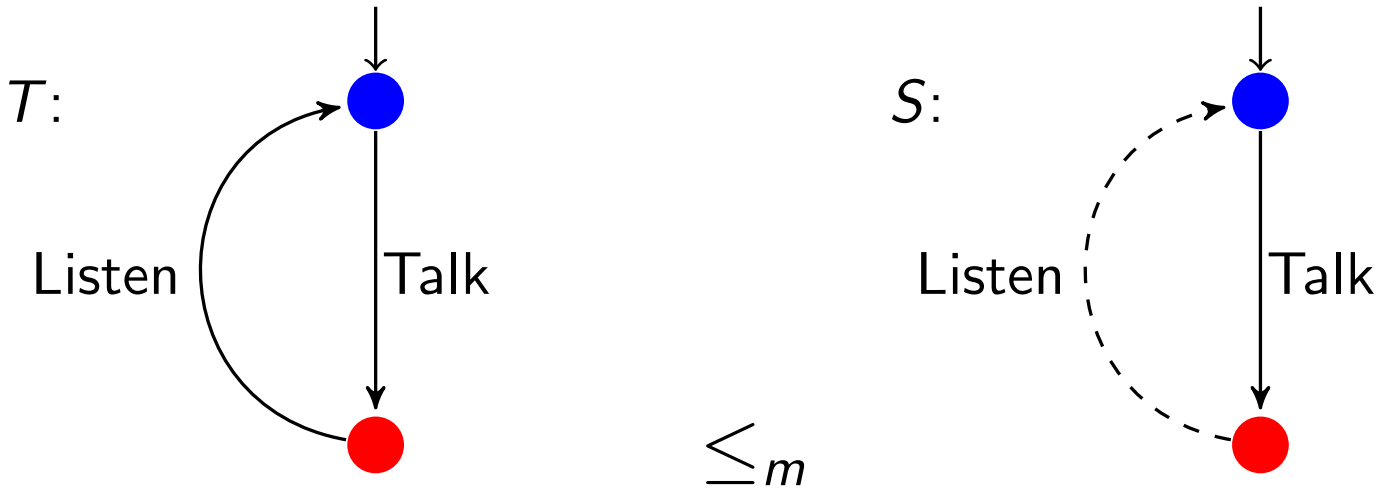

MTS Definition

Definition (Modal Transition System)

An MTS is a triple $(P, \dashrightarrow, \longrightarrow)$ where P is a set of states and $\longrightarrow \subseteq \dashrightarrow \subseteq P \times \Sigma \times P$.

If $S \xrightarrow{a} S'$ then also $S \overset{a}{\dashrightarrow} S'$; we draw only the must transition.

If $\longrightarrow = \dashrightarrow$ then the MTS is called an implementation.

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Definition (Modal Refinement $\leq_m$)

We write $T \leq_m S$ if there is a relation $\mathcal{R} \subseteq P \times P$ s.t. $(T, S) \in \mathcal{R}$ and for every $(A, B) \in \mathcal{R}$

- if $A \overset{a}{\dashrightarrow} A'$ then $B \overset{a}{\dashrightarrow} B'$ and $(A', B') \in \mathcal{R}$, and
- if $B \xrightarrow{a} B'$ then $A \xrightarrow{a} A'$ and $(A', B') \in \mathcal{R}$.

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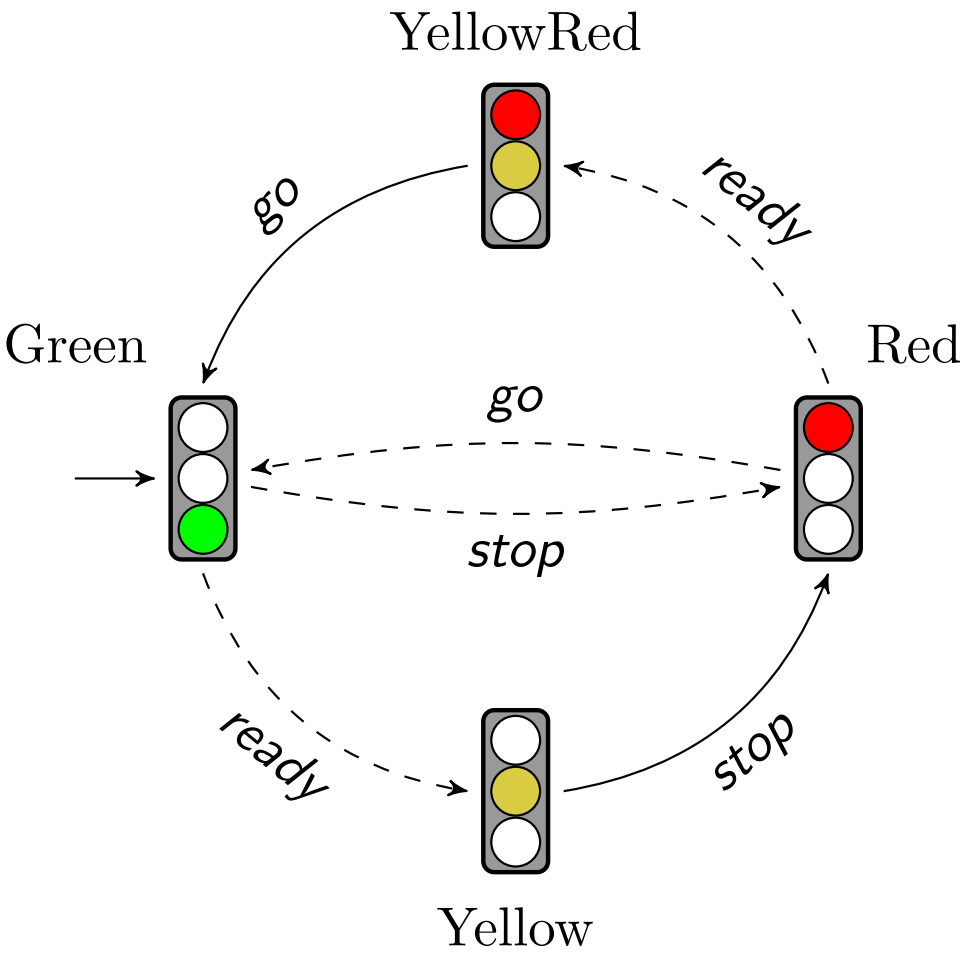
Note that on implementations $\leq_m$ coincides with bisimulation.

Parametric Modal Transition Systems [ATVA'11]

- MTS have many desirable properties but are a low level modelling formalism.
- Impossible to avoid deadlock states with outgoing may-transitions only.
- Impossible to define exclusive choices.
- Impossible to enforce persistent choices.

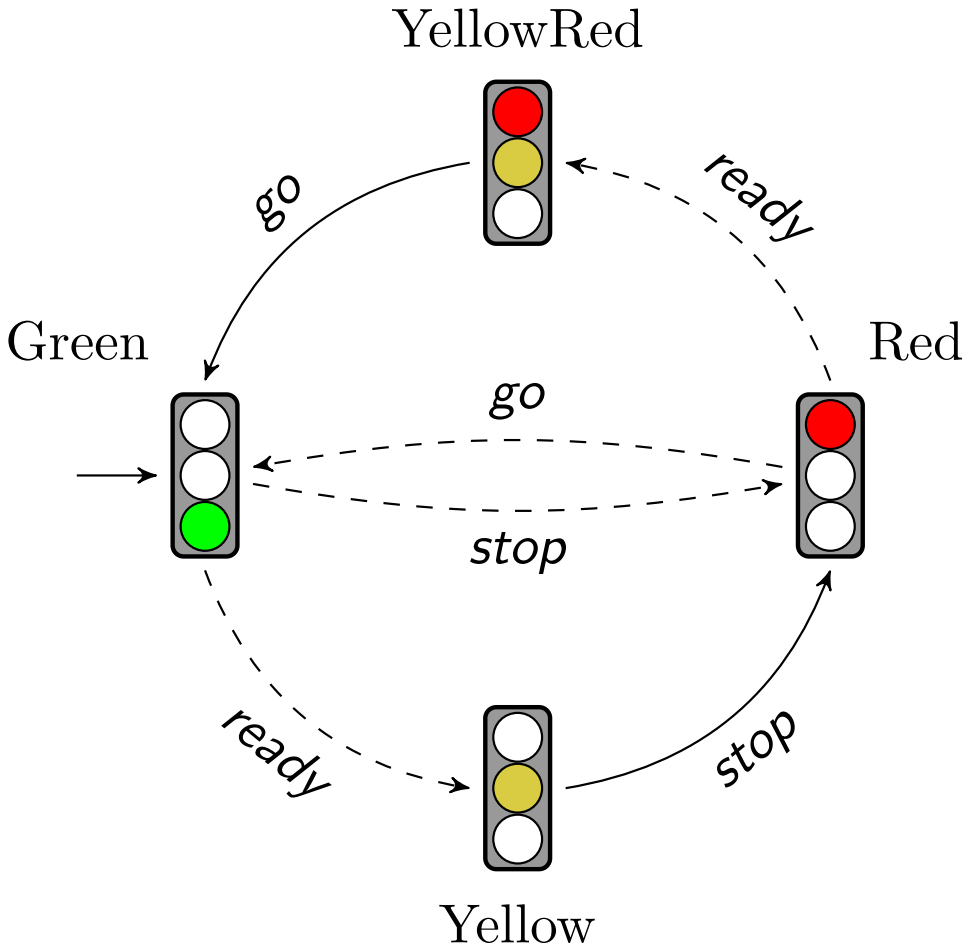
We suggest parametric MTS to deal with these issues.

Example: A Traffic Light

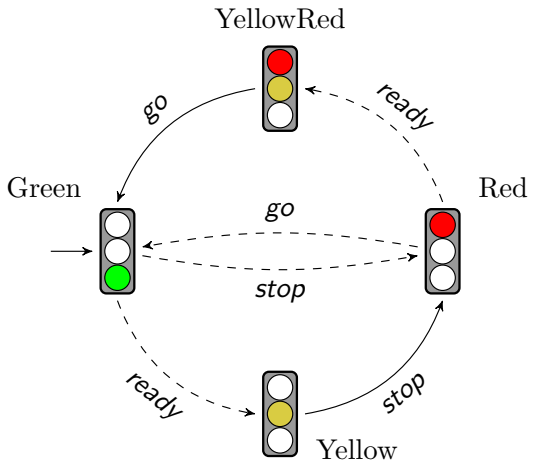


Example: A Traffic Light

$\leq m$



More Problematic Implementations



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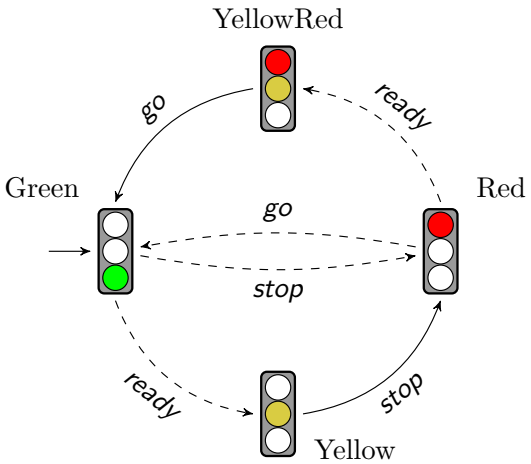

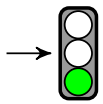

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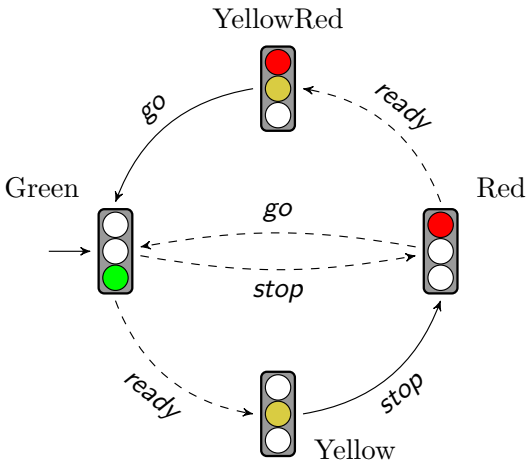

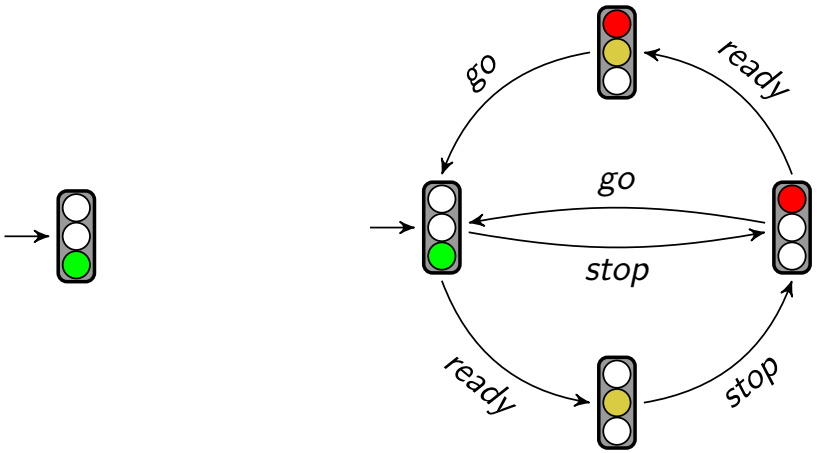

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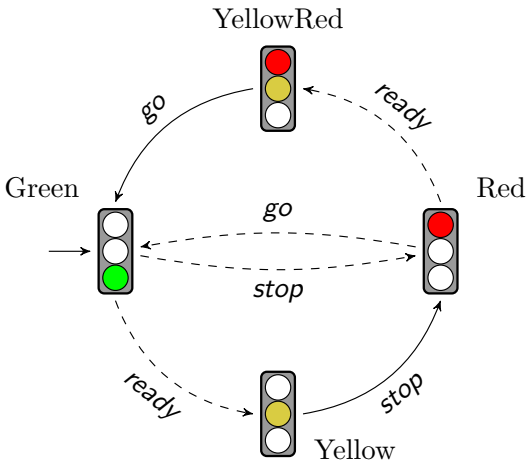
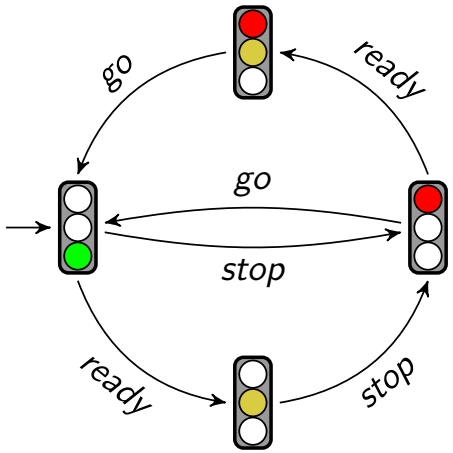
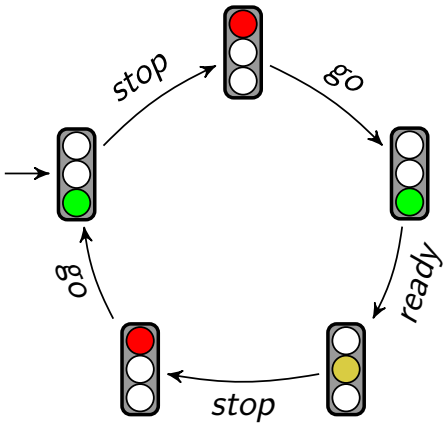
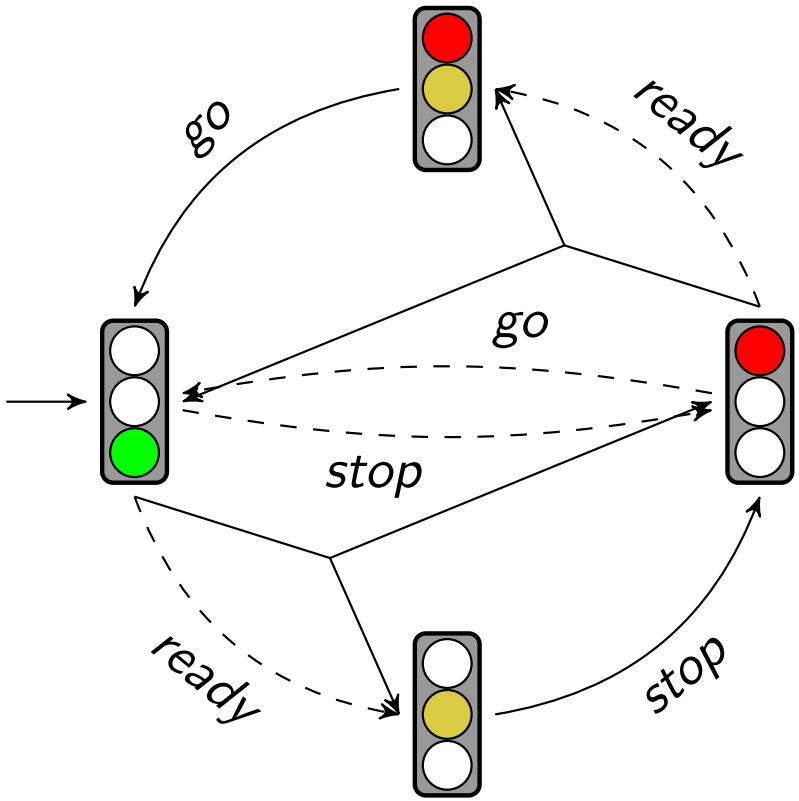






Disjunctive MTS



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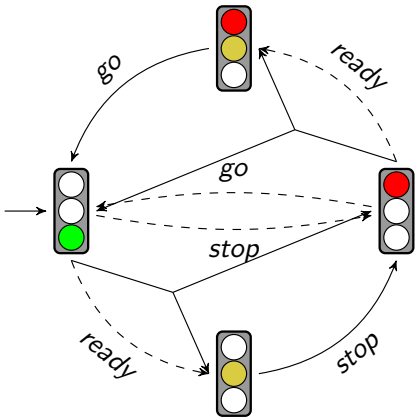
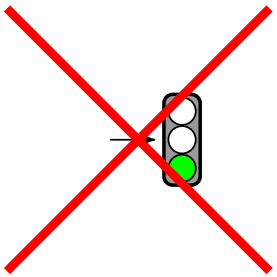
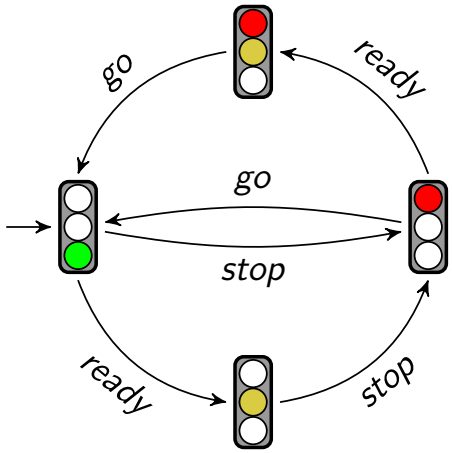
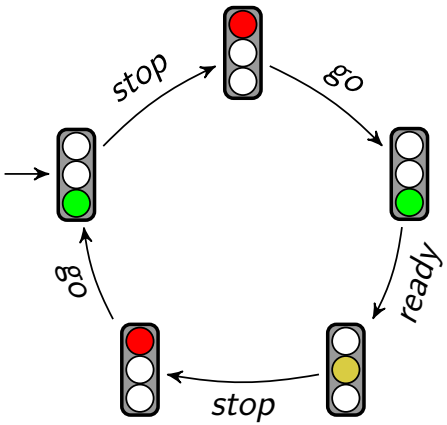
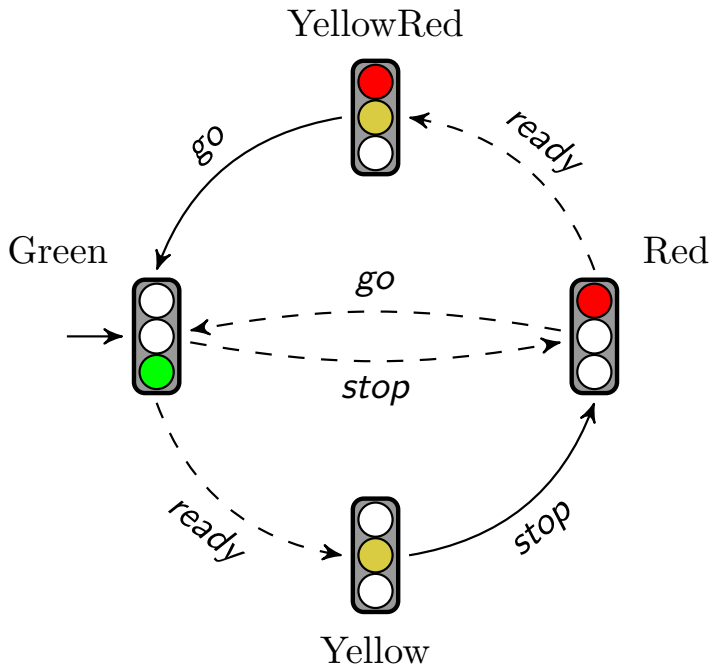








MTS with Obligations



Obligation function $\Phi : \text{States} \to \mathcal{B}(\text{Transitions})$

$$\begin{aligned}
\Phi(\text{Yellow}) &= \textit{stop} \\
\Phi(\text{YellowRed}) &= \textit{go} \\
\Phi(\text{Green}) &= \textit{stop} \oplus \textit{ready} \\
\Phi(\text{Red}) &= \textit{go} \oplus \textit{ready}
\end{aligned}$$

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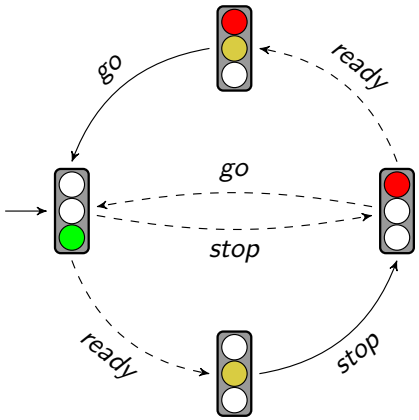

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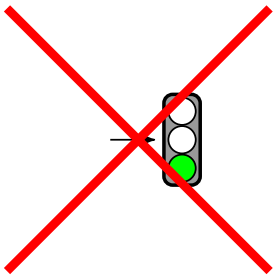
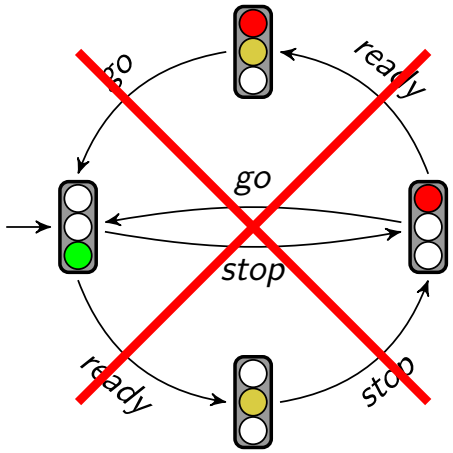
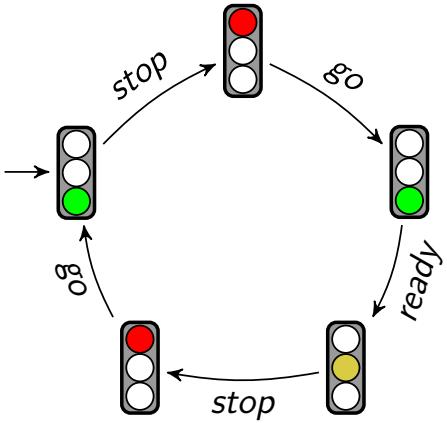
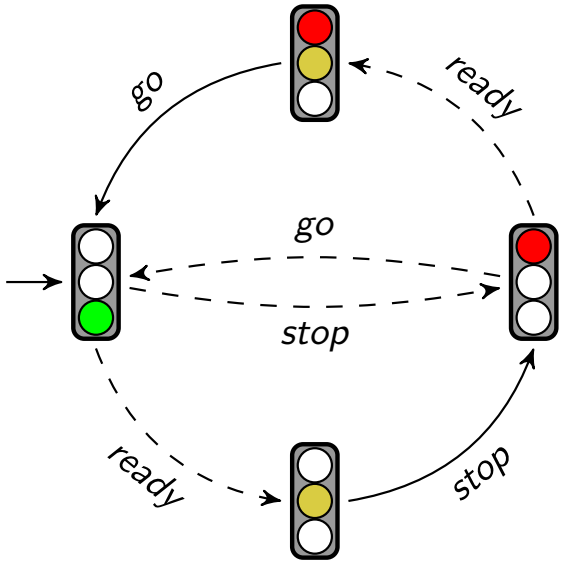






Parametric Modal Transition System



Obligation function $\Phi : \text{States} \rightarrow \mathcal{B}(\text{Transitions} \cup \text{Parameters})$

$\text{Parameters} = \{\mathit{RthenY}, \mathit{GthenY}\}$

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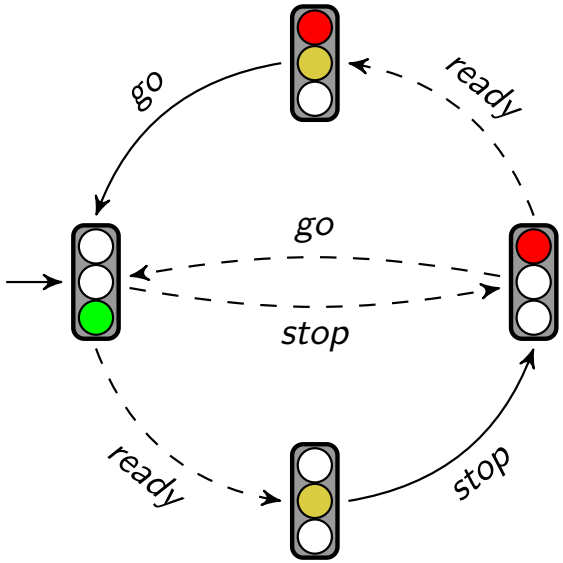

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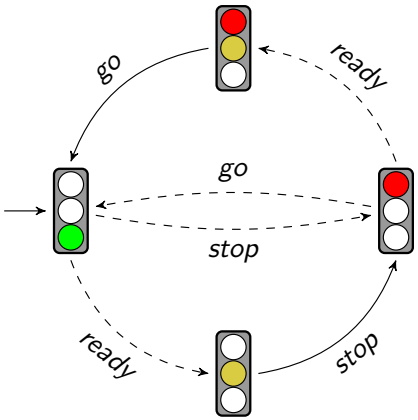

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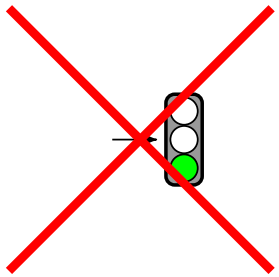
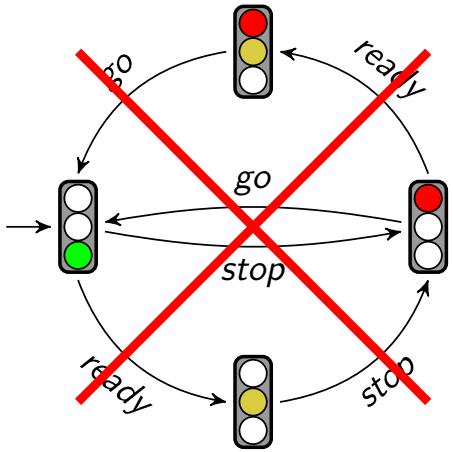
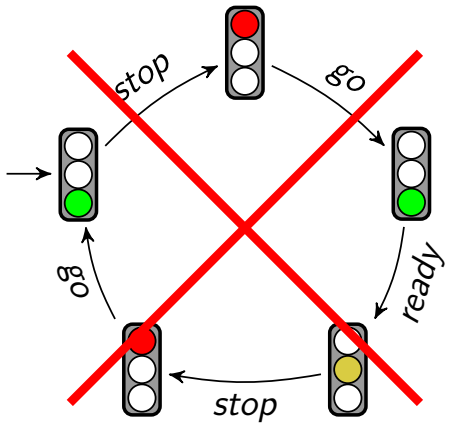
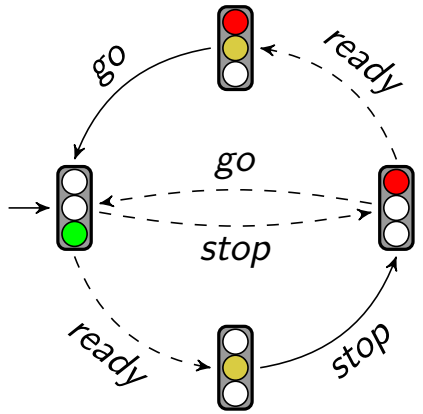






Modal Refinement

Parameters: $\{RthenY, GthenY\}$



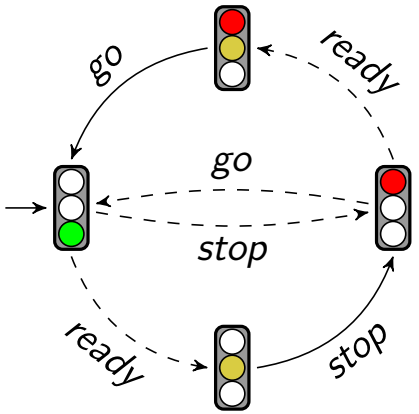
Obligation function:

$$\Phi(\mathrm{Green}) = (stop \oplus ready) \wedge (GthenY \Leftrightarrow ready)$$

$$\Phi(\mathrm{Red}) = (go \oplus ready) \wedge (RthenY \Leftrightarrow ready)$$

Modal Refinement

Parameters: $\{reqY\}$



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$\leq_m$

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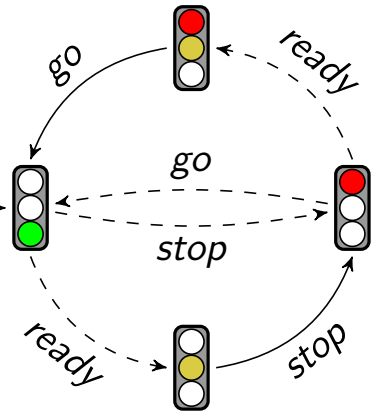

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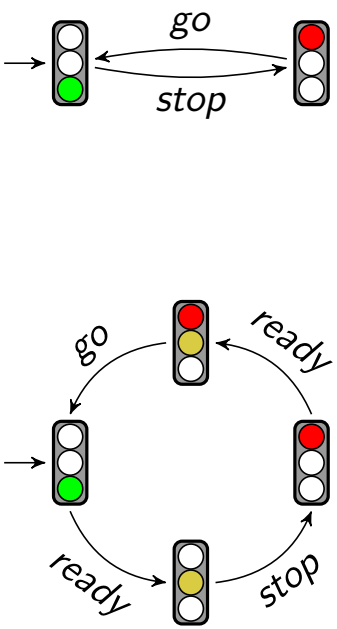

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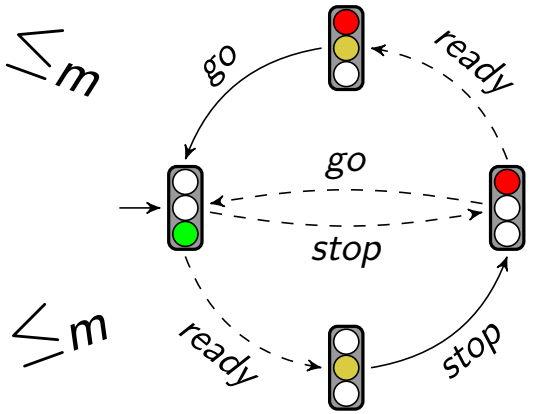
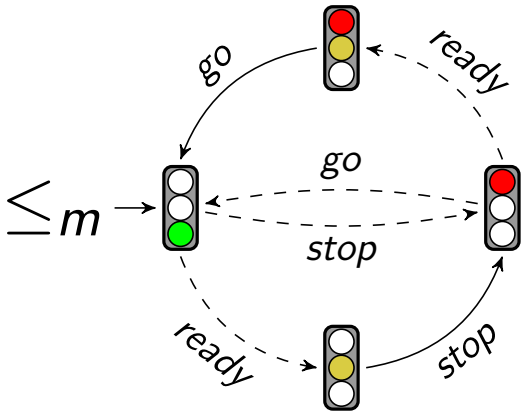


Obligation function:

$$\Phi(\mathrm{Green}) = (stop \oplus ready) \wedge (reqY \Leftrightarrow ready)$$

$$\Phi(\mathrm{Red}) = (go \oplus ready) \wedge (reqY \Leftrightarrow ready)$$

Parameters: $\{RthenY, GthenY\}$



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$$\Phi(\mathrm{Green}) = (stop \oplus ready) \wedge (GthenY \Leftrightarrow ready)$$

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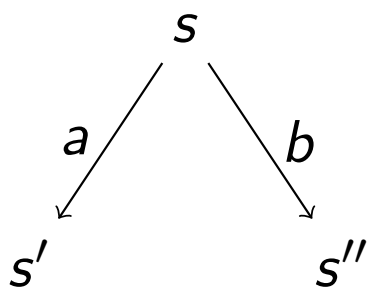
Modal Refinement

For $s \in \text{States}$ and $\nu \subseteq \text{Parameters}$, let

$$\text{PossibleTransitions}_{\nu}(s) = \{T \subseteq \text{Transitions}(s) \mid T \cup \nu \models \Phi(s)\}$$

Example:



For $\Phi(s) = a \Leftrightarrow p$,

$$\text{PossibleTransitions}_{\{p\}}(s) = \Big\{\{a\}, \{a, b\}\Big\}$$

Modal Refinement

For $s \in \text{States}$ and $\nu \subseteq \text{Parameters}$, let

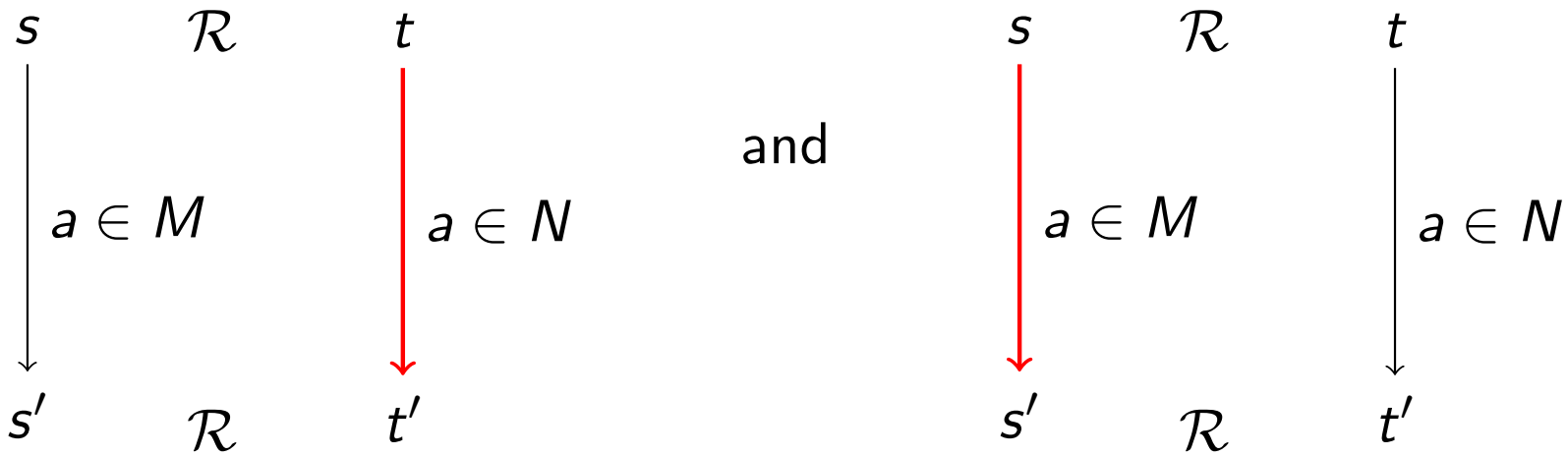
$$\text{PossibleTransitions}_\nu(s) = \{T \subseteq \text{Transitions}(s) \mid T \cup \nu \models \Phi(s)\}$$

Definition (Modal Refinement)

Binary relation $\mathcal{R}$ is a *modal refinement* if

$\forall \mu \subseteq \text{Parameters}_1 : \exists \nu \subseteq \text{Parameters}_2 :$ for every $(s,t) \in \mathcal{R}$:

$\forall M \in \text{PossibleTransitions}_\mu(s) : \exists N \in \text{PossibleTransitions}_\nu(t)$:



Complexity Results of Modal Refinement Checking

Parameter-free PMTS:

	Boolean	Positive	pCNF	pDNF	MTS
Boolean	Π_2^{p}-comp.	coNP-comp.	$\in$ coNP P-hard	coNP-comp.	$\in$ coNP P-hard
Positive	Π_2^{p}-comp.	coNP-comp.	P-comp.	coNP-comp.	P-comp.
pCNF	Π_2^{p}-comp.	coNP-comp.	P-comp.	coNP-comp.	P-comp.
pDNF	Π_2^{p}-comp.	P-comp.	P-comp.	P-comp.	P-comp.
MTS	Π_2^{p}-comp.	P-comp.	P-comp.	P-comp.	P-comp.
Impl	NP-comp.	P-comp.	P-comp.	P-comp.	P-comp.

Complexity Results of Modal Refinement Checking

Parameter-free PMTS:

	Boolean	Positive	pCNF	pDNF	MTS
Boolean	Π_2^{p}-comp.	coNP-comp.	$\in$ coNP P-hard	coNP-comp.	$\in$ coNP P-hard
Positive	Π_2^{p}-comp.	coNP-comp.	P-comp.	coNP-comp.	P-comp.
pCNF	Π_2^{p}-comp.	coNP-comp.	P-comp.	coNP-comp.	P-comp.
pDNF	Π_2^{p}-comp.	P-comp.	P-comp.	P-comp.	P-comp.
MTS	Π_2^{p}-comp.	P-comp.	P-comp.	P-comp.	P-comp.
Impl	NP-comp.	P-comp.	P-comp.	P-comp.	P-comp.

(General) PMTS:

	Boolean	positive	pCNF	pDNF
Boolean	Π_4^{p}-comp.	Π_3^{p}-comp.	$\in \Pi_3^{\mathrm{p}}$ Π_2^{p}-hard	Π_3^{p}-comp.
positive	Π_4^{p}-comp.	Π_3^{p}-comp.	Π_2^{p}-comp.	Π_3^{p}-comp.
pCNF	Π_4^{p}-comp.	Π_3^{p}-comp.	Π_2^{p}-comp.	Π_3^{p}-comp.
pDNF	Π_4^{p}-comp.	Π_2^{p}-comp.	Π_2^{p}-comp.	Π_2^{p}-comp.
MTS	Σ_3^{p}-comp.	NP-comp.	NP-comp.	NP-comp.
Impl	NP-comp.	NP-comp.	NP-comp.	NP-comp.

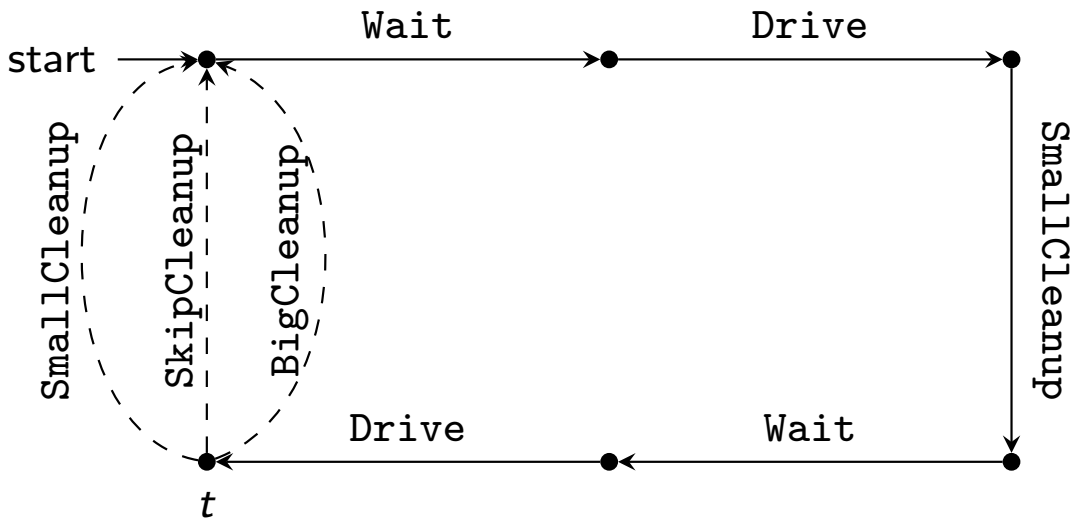
Cost and Duration [LPAR'12]

The cost of implemantations matters! (product lines)

- We suggest a quantitative extension of parametric MTS with cost and durations.
- Dual-priced scheme (investment cost plus the running cost).

Modal Transition Systems with Durations

$$\Phi(t) = \begin{array}{l} \texttt{BigCleanup} \vee \\ \texttt{SkipCleanup} \vee \\ \texttt{SmallCleanup} \end{array}$$



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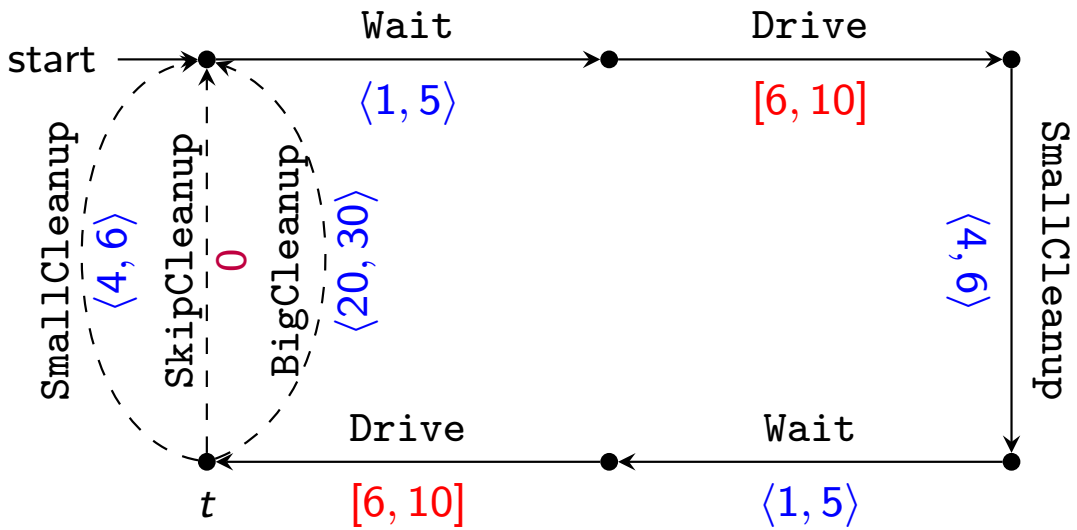

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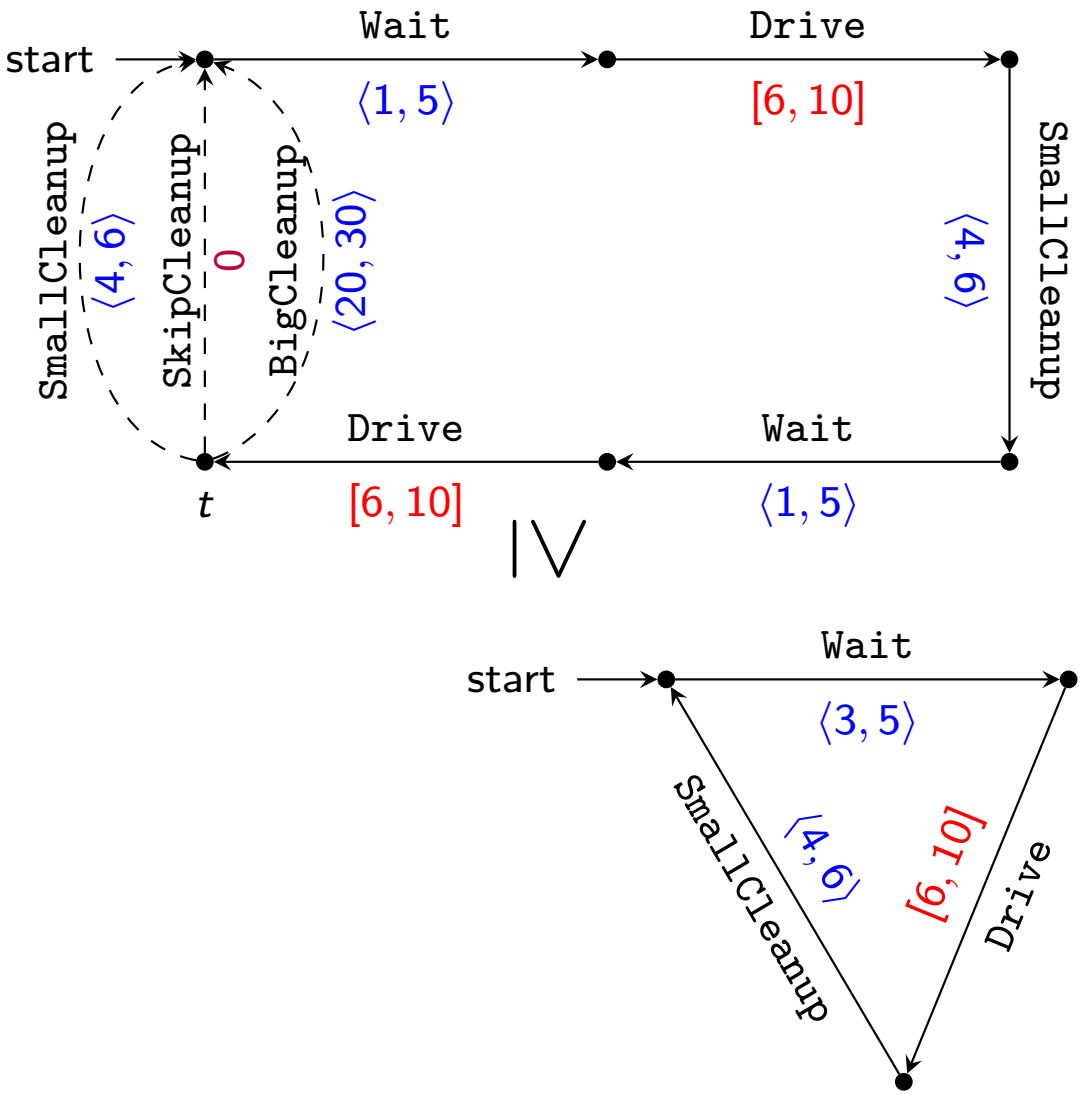

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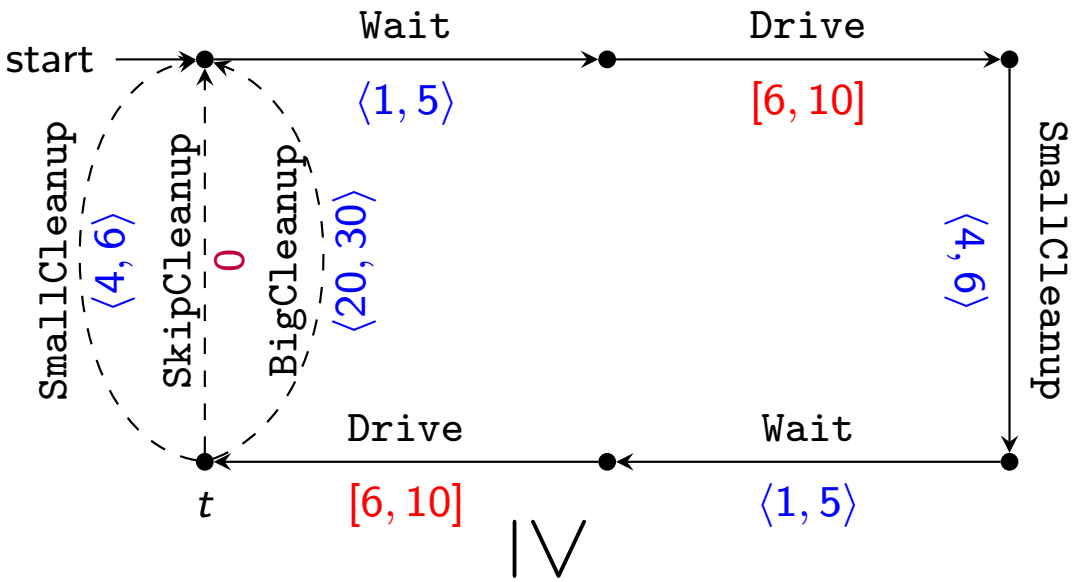
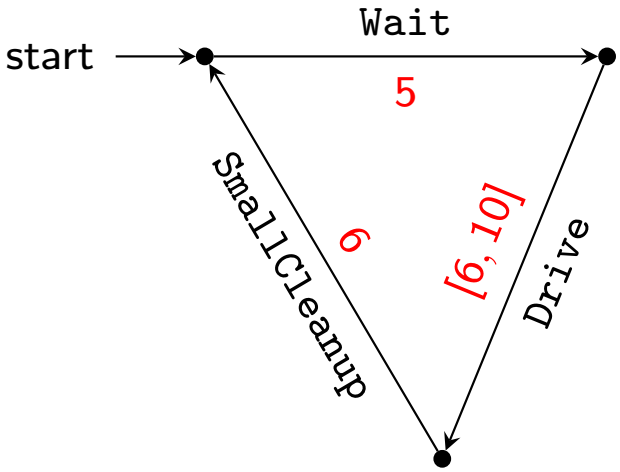
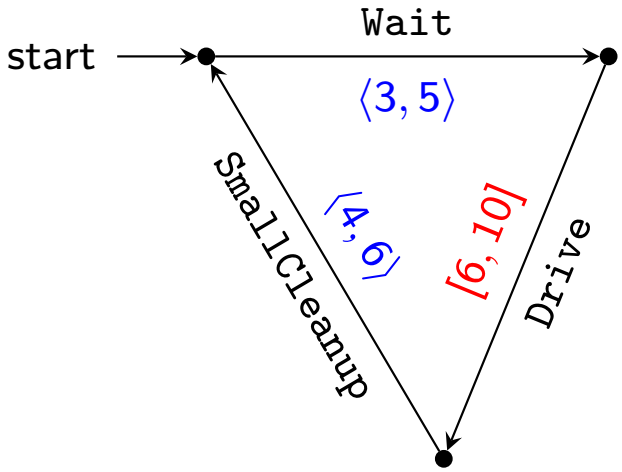


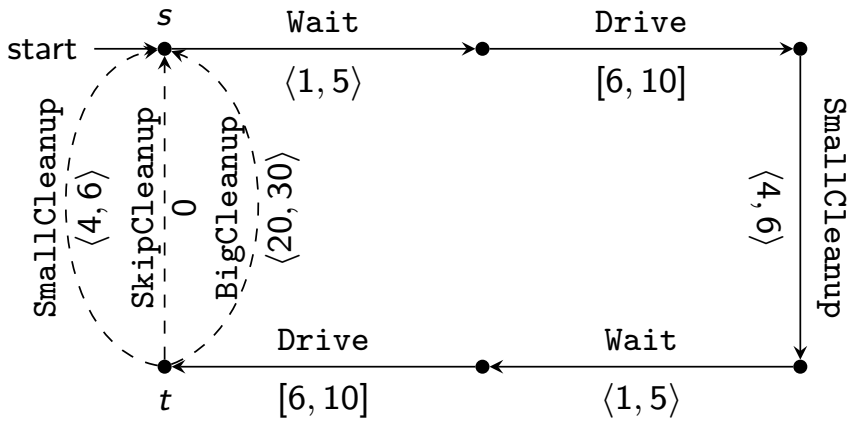


$\preceq_m$



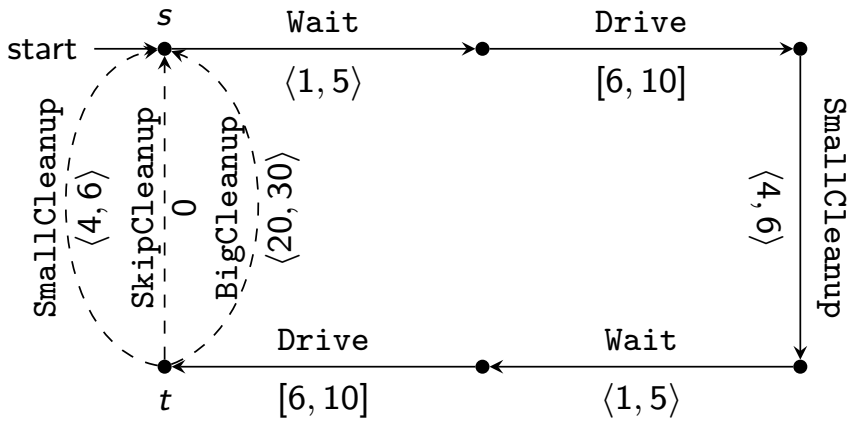
Dual-Price Scheme

$\Phi(t) =$ `BigCleanup` $\vee$
`SkipCleanup` $\vee$
`SmallCleanup`



Dual-Price Scheme

$\Phi(t) =$ BigCleanup $\vee$ SkipCleanup $\vee$ SmallCleanup



Hardware:
$H = \{\texttt{VacuumCleaner}, \texttt{Sponge}\}$

Requirements:
$\Psi(\texttt{BigCleanup}) = \texttt{VacuumCleaner}$
$\Psi(\texttt{SmallCleanup}) = \texttt{VacuumCleaner} \vee \texttt{Sponge}$

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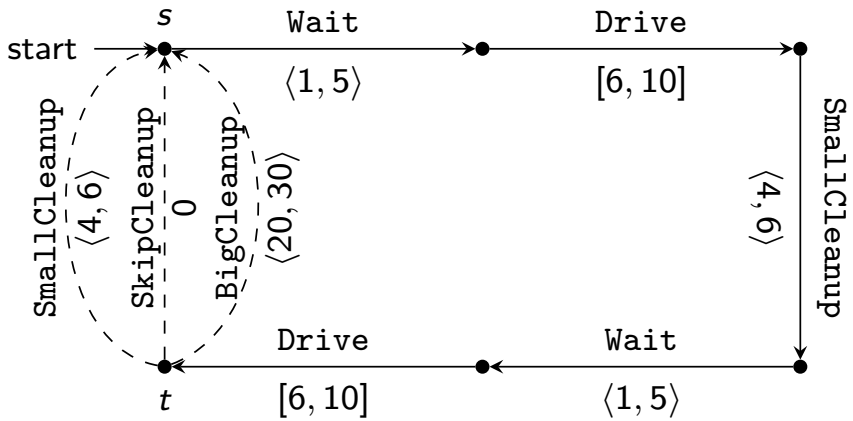

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Investment:
$i(\texttt{VacuumCleaner}) = 100$
$i(\texttt{Sponge}) = 5$

Dual-Price Scheme

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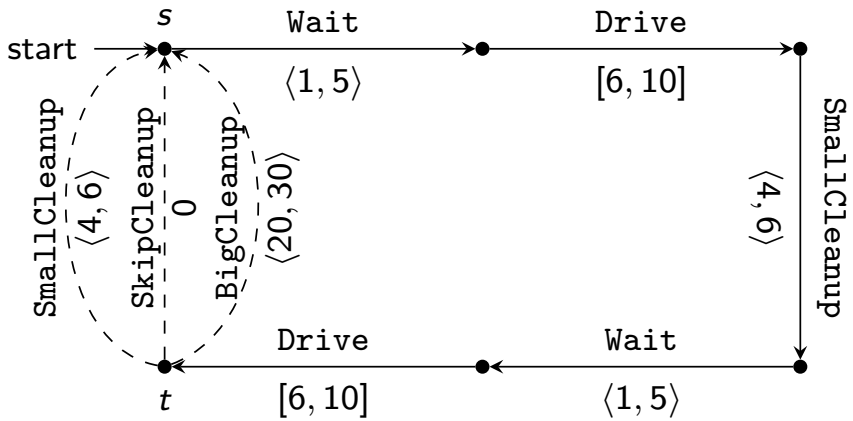

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Investment:
$i(\texttt{VacuumCleaner}) = 100$
$i(\texttt{Sponge}) = 5$

Running Cost:

$a \in \Sigma$	$r(a)$
Wait	8
Drive	10
SmallCleanup	6
BigCleanup	7
SkipCleanup	0

Investment Cost

Definition (Investment Cost)

$$\mathrm{ic}(\mathcal{I}) = \min_{G \models \mathcal{I}} \sum_{g \in G} i(g)$$

Requirements:

$\Psi(\texttt{BigCleanup}) = \texttt{VacuumCleaner}$

$\Psi(\texttt{SmallCleanup}) = \texttt{VacuumCleaner} \vee \texttt{Sponge}$

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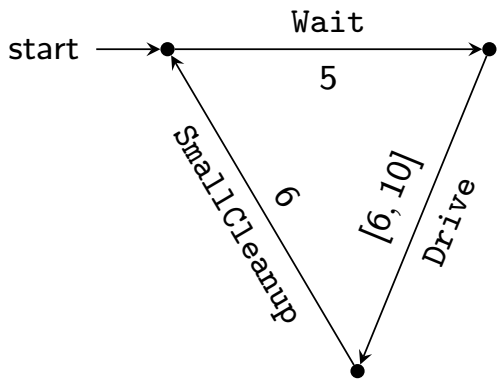

$$\mathrm{ic}(\mathcal{I}_1) = 5$$

Investment Cost

Definition (Investment Cost)

$$\mathrm{ic}(\mathcal{I}) = \min_{G \models \mathcal{I}} \sum_{g \in G} i(g)$$

Requirements:

$\Psi(\texttt{BigCleanup}) = \texttt{VacuumCleaner}$

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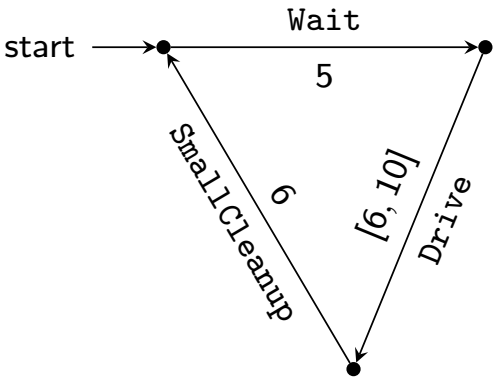

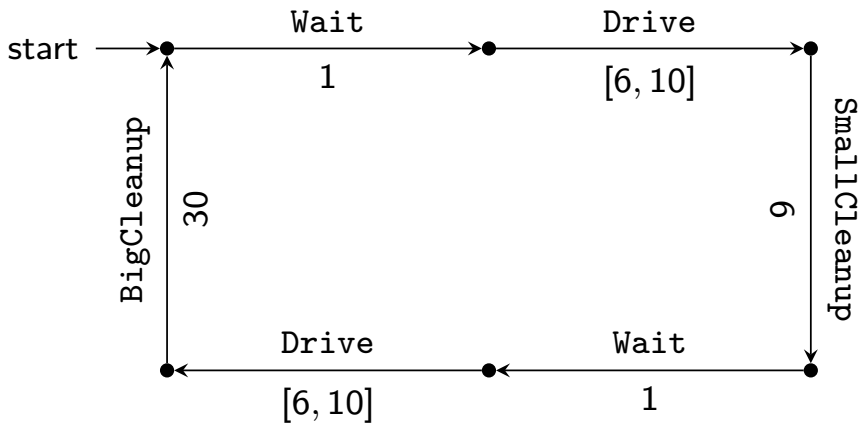

$\mathrm{ic}(\mathcal{I}_1) = 5$

$\mathrm{ic}(\mathcal{I}_2) = 100$

Running Cost

Definition (Running Cost)

$$\mathrm{rc}(\mathcal{I}) = \sup_{s_0 a_0 t_0 s_1 a_1 t_1 \cdots \in \mathcal{R}un(\mathcal{I})} \limsup_{n \to \infty} \frac{\sum_{i=0}^{n} r(a_i) \cdot t_i}{\sum_{i=0}^{n} t_i}$$

Running Cost

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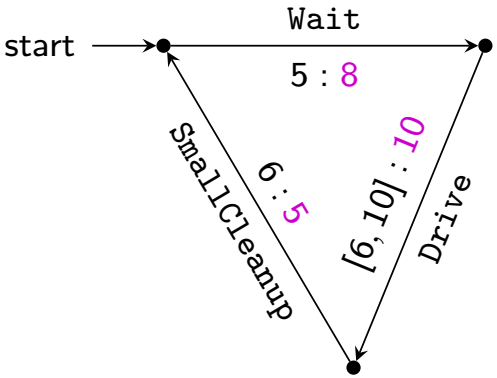

$$\frac{5 \cdot 8 + 10 \cdot 10 + 6 \cdot 5}{5 + 10 + 6}$$

$$\approx 8.10$$

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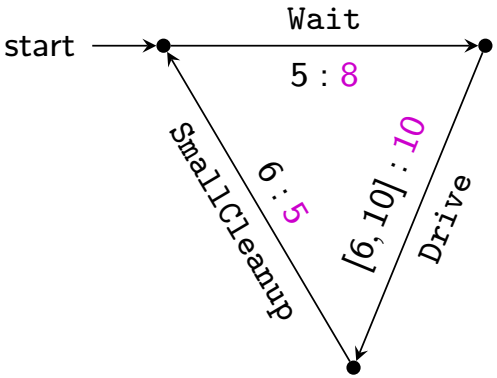

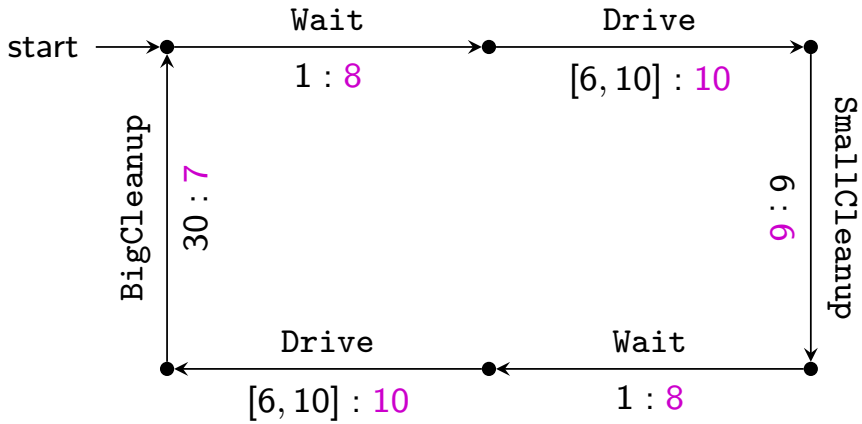

$$\frac{5 \cdot 8 + 10 \cdot 10 + 6 \cdot 5}{5 + 10 + 6}$$

$$\approx 8.10$$

$$\frac{1 \cdot 8 + 10 \cdot 10 + 6 \cdot 6 + 1 \cdot 8 + 10 \cdot 10 + 30 \cdot 7}{1 + 10 + 6 + 1 + 10 + 30}$$

$$\approx 7.97$$

Main Results

Problem (The Implementation Problem)

Given

- an MTSD specification $\mathcal{S}$,
- a dual-price scheme,
- an upper-bound max_{ic} for the investment cost and
- an upper bound max_{rc} on the running cost,

does there exist an implementation $\mathcal{I} \leq_m \mathcal{S}$ such that

- $\text{ic}(\mathcal{I}) \leq max_{\text{ic}}$ and
- $\text{rc}(\mathcal{I}) \leq max_{\text{rc}}$?

Main Results

Problem (The Implementation Problem)

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does there exist an implementation $\mathcal{I} \leq_m \mathcal{S}$ such that

- $\text{ic}(\mathcal{I}) \leq max_{\text{ic}}$ and
- $\text{rc}(\mathcal{I}) \leq max_{\text{rc}}$?

Theorem

The implementation problem is NP-complete.

Main Results

Theorem

The implementation problem with

- positive obligation function Φ and
- a constant number of hardware components H

is polynomially equivalent to mean payoff games and thus it is in NP $\cap$ coNP and solvable in pseudo-polynomial time.

Conclusion

Modal transition systems are an elegant formalism for modelling of high-level specifications of reactive systems.

Our extension with parameters:

- specializes to refinements on the well-studied subclasses, and
- allows to model persistent choices.

Our dual-priced extension:

- shows how to deal with quantitative aspects,
- combines investment and running cost.

Future Work:

- Further generalization of the formalism to non-flat systems.
- Tool integration, case studies, application to product lines.